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Analytical Prediction of Quench Energies of Cooled Superconductors Based on the Hyperbolic Heat Conduction Model

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Abstract The critical energy characteristics of cooled composite superconductors is analytically predicted based on the one-dimensional hyperbolic heat conduction model. The temperature dependence of the Ohmic heat generation, the finite speed of heat transfer, and the finite duration and finite length of the thermal disturbances are taken into account in the present model. The critical energies are calculated using a model based on the analytical solution of the hyperbolic heat conduction equation by the Laplace transformation method. The computational model results show that the critical energy depends on the relaxation time and disturbance duration. It is found that the hyperbolic conduction model predicts a lower-critical energy as compared to the predictions of the parabolic heat conduction model.

Keywords Superconductors · Critical energy · Thermal stability · Hyperbolic heat conduction · Non-Fourier heat conduction · Laplace transform

1 Introduction

The minimum amount of energy that triggers quenching for a superconductor is called the critical energy, and it is an indication of the thermal stability of superconducting magnets. The superconductor's critical energy depends on many factors, such as cooling conditions, current density, magnetic field, thermal properties of the conductor, heat disturbance duration and length, etc. If the disturbance energy does not exceed the critical energy, quenching will never occur, and the superconductor is said to be thermally stable. Otherwise, a local normal zone can be initiated;

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this normal zone may grow or shrink depending on the operating conditions of the superconductor. The normal zone characteristics are investigated via critical energy predictions based on the analysis of the transient temperature field in the normal zone.

Due to its important application in engineering technologies, such as nuclear fusion, superconducting switches, fault-current limiters, cellular phone base stations, and power distribution networks, the problem of critical energy prediction has been investigated by many authors [1–11]. Most previous studies were based on the parabolic (classical diffusion) heat conduction model which assumes an infinite speed of heat transport in the composite conductor, to compute the critical energy of cooled or uncooled superconductors [1–10]. Very few of them have considered the hyperbolic conduction model to study the thermal stability of superconductors using numerical and analytical solution methods [12–15]. The use of an analytical method to calculate the critical energy of uncooled superconductors under the effect of the one-dimensional hyperbolic heat conduction was carried out by Lewandowska and Malinowski [16]. In most practical superconducting applications, the classical Fourier law (parabolic equation) of heat conduction results are adequate. However, there are many applications where the use of the hyperbolic model becomes necessary, such as ultra-fast heat conduction processes, heat conduction at cryogenic temperatures, and cases where the time of interest is very short and when the heat source in the conductor depends on temperature and/or time [17–20]. In operation, superconductors are subjected to all of these circumstances to some extent. Under these circumstances, the heat propagation speed can be finite.

Based on the authors' best knowledge, an analytical model to predict the critical energy of cooled superconductors based on the hyperbolic heat conduction model has not been yet reported. This is the main motivation of the present study which is devoted to investigate analytically the characteristics of the critical energy of cooled superconductors based on the hyperbolic heat conduction model. In order to predict the critical energy, the hyperbolic heat condition equations are solved analytically by means of a Laplace transformation technique. In the present model, the finite duration and finite length of thermal disturbances are taken into consideration. The critical energies obtained from the hyperbolic model are compared with those predicted by the parabolic model. The effects of various parameters on the cooled superconductor critical energy were analyzed in detail.

2 Problem Formulation and Analysis

Consider a very long composite superconductor of diameter (d) carrying an electrical current perpendicular to the x-y plane and subjected to a thermal disturbance as shown in Fig. 1. The composite superconductor consists of superconducting strands (filaments) embedded in a high purity metal. The superconductor is laterally cooled using a cooling liquid having a convective heat transfer coefficient h. The superconductor is subjected to an external thermal disturbance of finite length and finite duration. The temperature fields in the superconductor composite and the quenching process are governed by the energy equation coupled with the hyperbolic heat conduction

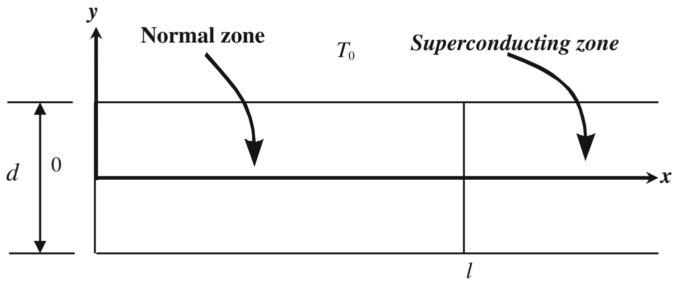

Fig. 1 Schematic diagram for the problem under consideration

constitution law. With the assumption of constant and isotropic thermal properties, the governing equations are given as

$$C\frac{\partial T(x,t)}{\partial t} = k\nabla \cdot \mathbf{q} - \frac{P}{A}\left[h(T - T_0) + g(T) + D(x,t)\right] \tag{1}$$

$$\mathbf{q}(x,t) + t_q\frac{\partial \mathbf{q}(x,t)}{\partial t} = -k\nabla T(x,t) \tag{2}$$

Eliminating the heat flux vector **q** between Eqs. 1 and 2 leads to the hyperbolic heat conduction equation describing the temperature field in the superconducting composite as follows:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{t_q}{\alpha}\frac{\partial^2 T}{\partial t^2} = \frac{\partial^2 T}{\partial x^2} + \frac{P}{kA}\left[g(T) + t_q\frac{\partial g(T)}{\partial t} + D(T) + t_q\frac{\partial D}{\partial t} - Q(T) - t_q\frac{\partial Q(T)}{\partial t}\right] \tag{3}$$

where t_q is the relaxation time which represents the delay of the heat flux with respect to the change in temperature gradient, α is the thermal diffusivity, k is the thermal conductivity, P is the wetted perimeter, A is the cross-sectional area of the conductor, $g(x,t)$ is the Ohmic heat generation, $D(x,t)$ is the external thermal disturbance energy, and $Q(T)$ is the heat transfer to the coolant. $Q(T) = h(T - T_0)$. Equation 3 describes the behavior of a damped temperature wave with a finite speed equal to $w = [\alpha/t_q]^{0.5}$. When $t_q \to 0$ and $w \to \infty$, Eq. 3 reduces to the classical parabolic (Fourier law) equation of heat conduction. The thermal disturbance energy is simulated as a rectangular heat pulse in x and t, as follows:

$$D(x,t) = \begin{cases} \frac{E}{2plt_i} & \text{for } -l < x < l \text{ and } 0 < t < t_i \\ 0 & \text{outside this region} \end{cases} \tag{4}$$

The steady capacity of the Ohmic heat generation in the conductor is calculated based on the current sharing model [3] as

$$g(T) = \begin{cases} 0 & \text{for} \quad T \le T_{\text{cs}} \\ \frac{g\max}{f}\frac{T-T_{\text{cs}}}{T_{\text{c}}-T_{\text{cs}}} & \text{for} \quad T_{\text{cs}} < T < T_{\text{c}} \\ \frac{g\max}{f} & \text{for} \quad T \ge T_{\text{c}} \end{cases} \tag{5}$$

where $g_{\max} = \rho^2 J/fP$ is the Ohmic heat generation with the entire current in the stabilizer. The current sharing function $g(T)$ given in Eq. 5 is based on the assumption that the critical current density of the superconductor is a linear function of temperature at a fixed magnetic field. In order to obtain an analytical solution to Eq. 3, the Ohmic heat term shoud be linearized as

$$g(T) = a(T - T_{\text{o}}) \tag{6a}$$

where

$$a = \frac{g_{\max}}{(T_{\text{cs}} + T_{\text{c}} - 2T_{\text{o}})} \tag{6b}$$

$$T_{\text{cs}} = T_{\text{c}} - \frac{I}{I_{\text{c}}}(T_{\text{c}} - T_{\text{o}}) \tag{6c}$$

The initial and boundary conditions are

$$T(x, 0) = T_{\text{o}}, \frac{\partial T(x, y, 0)}{\partial t} = \dot{T}_i \tag{7}$$

$$T(-l, t) = T(-l, t) = T_0, \tag{8}$$

It is more convenient to rewrite Eqs. 3–8 using the following dimensionless parameters:

$$\left.\begin{array}{l} \xi = \dfrac{x}{l}, \tau = \dfrac{\alpha t}{l^2}, \theta = \dfrac{T - T_0}{T_{\text{c}} - T_0}, \\ Bi = \dfrac{hPl^2}{Ak}, G = \dfrac{Pl^2 g_{\max}}{Ak(T_{\text{c}} - T_0)}, \phi = \dfrac{E}{2AlC(T_{\text{c}} - T_0)} \end{array}\right\} \tag{9}$$

The dimensionless form of the governing equation can be written in terms of the dimensionless parameters of Eq. 9 as

$$\begin{aligned} \tau_q \frac{\partial^2 \theta}{\partial \tau^2} + \frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \xi^2} + \left(\frac{G}{1 + \theta_{\text{c}}} - Bi\right)\left(\theta + \tau_q \frac{\partial \theta}{\partial \tau}\right) \\ + \frac{\varphi}{\tau_i}\left(u(\xi, \tau) + \tau_q \frac{\partial u(\xi, \tau)}{\partial \tau}\right) \end{aligned} \tag{10}$$

where

$$u(\xi,\tau)=\begin{cases}1 & \text{for} -1\le\xi\le 1 \text{ and } 0\le\tau\le\tau_i\\ 0 & \text{outside this region}\end{cases}\tag{11}$$

Due to the symmetry of the normal zone, the analysis is limited to the half zone, i.e., to the domain that lies within $0\ge x\ge l$. Equation 10 can be solved separately for the normal part of the conductor that is subjected to the heat disturbance, and for the superconducting part. This results in the following two equations:

$$\tau_q\frac{\partial^2\theta_1}{\partial\tau^2}+\frac{\partial\theta_1}{\partial\tau}=\frac{\partial^2\theta_1}{\partial\xi^2}+\left(\frac{G}{1+\theta_c}-Bi\right)\left(\theta_1+\tau_q\frac{\partial\theta_1}{\partial\tau}\right)+\frac{\varphi}{\tau_i}\left(u(\tau)+\tau_q\frac{\partial u(\tau)}{\partial\tau}\right)\tag{12}$$

$$\tau_q\frac{\partial^2\theta_2}{\partial\tau^2}+\frac{\partial\theta_2}{\partial\tau}=\frac{\partial^2\theta_2}{\partial\xi^2}+\left(\frac{G}{1+\theta_c}-Bi\right)\left(\theta_2+\tau_q\frac{\partial\theta_2}{\partial\tau}\right)\tag{13}$$

with the following initial and boundary conditions,

$$\left.\begin{array}{ll}\theta_1(\xi,0)=0, & \theta_2(\xi,0)=0\\ \dfrac{\partial\theta_1(\xi,0)}{\partial\tau}=\dfrac{\phi}{\tau_i} & \dfrac{\partial\theta_2(\xi,0)}{\partial\tau}=0\end{array}\right\}\tag{14}$$

$$\left.\begin{array}{ll}\dfrac{\partial\theta_1(0,\tau)}{\partial\tau}=0, & \dfrac{\partial\theta_1(1,\tau)}{\partial\tau}=\dfrac{\partial\theta_2(1,\tau)}{\partial\tau}\\ \theta_1(1,\tau)=\theta_2(1,\tau) & \theta_2(\infty,\tau)=0\end{array}\right\}\tag{15}$$

where

$$u(\tau)=\begin{cases}1 & \text{for } 0\le\tau\le\tau_i\\ 0 & \text{outside this region}\end{cases}\tag{16}$$

2.1 Solution Methodology

The governing equations are solved analytically by means of the Laplace transformation technique. Taking the Laplace transform of Eqs. 11 and 12 and utilizing the initial conditions of Eq. 13 gives

$$\frac{d^2\bar\theta_1}{d\xi^2}-\left(\tau_q s^2+s-\left(\frac{G}{1+\theta_c}-Bi\right)(1+\tau_q s)\right)\bar\theta_1=-\frac{\varphi}{\tau_i}\left(\bar u(s)(1+\tau_q s)\right)\tag{17}$$

$$\frac{\mathrm{d}^2\bar{\theta}_2}{\mathrm{d}\xi^2} - \left(\tau_q s^2 + s - \left(\frac{G}{1+\theta_\mathrm{c}} - Bi\right)(1+\tau_q s)\right)\bar{\theta}_2 = 0 \tag{18}$$

where $\bar{\theta}_2(\xi, s) = L\left[\theta(\xi, \tau)\right]$.

The Laplace transformation of the boundary conditions are

$$\left.\begin{array}{ll} \dfrac{\mathrm{d}\bar{\theta}_1(0,s)}{\mathrm{d}\xi} = 0, & \bar{\theta}_2(\infty, s) = 0 \\ \bar{\theta}_1(1,s) = \bar{\theta}_2(1,s) & \dfrac{\mathrm{d}\bar{\theta}_1(1,s)}{\mathrm{d}\xi} = \dfrac{\mathrm{d}\bar{\theta}_2(1,s)}{\mathrm{d}\xi} \end{array}\right\} \tag{19}$$

The solution of the ordinary differential Eqs. 17 and 18 subjected to the boundary conditions of Eq. 19 are

$$\begin{aligned} \bar{\theta}_1(\xi, s) = {} & \frac{\varphi \bar{u}(s)}{2\tau_i\left(s - \left(\frac{G}{1+\theta_\mathrm{c}} - Bi\right)\right)} \\ & - \left\{2 - \exp\left[-(1-\xi)\sqrt{\psi(s)}\right] - \exp\left[-(1+\xi)\sqrt{\psi(s)}\right]\right\} \end{aligned} \tag{20}$$

$$\begin{aligned} \bar{\theta}_2(\xi, s) = {} & \frac{\varphi \bar{u}(s)}{2\tau_i\left(s - \left(\frac{G}{1+\theta_\mathrm{c}} - Bi\right)\right)} \\ & - \left\{\exp\left[(1-\xi)\sqrt{\psi(s)}\right] - \exp\left[-(1+\xi)\sqrt{\psi(s)}\right]\right\} \end{aligned} \tag{21}$$

where $\psi(s) = \left(1 + \tau_q s\right)\left(s - \left(\frac{G}{1+\theta_\mathrm{c}} - Bi\right)\right)$.

The critical energy is calculated by analyzing the maximum temperature in the conductor that occurs at the center of the normal zone (i.e., $\xi = 0$).

$$\bar{\theta}_1(0, s) = \frac{\varphi \bar{u}(s)}{2\tau_i\left(s - \left(\frac{G}{1+\theta_\mathrm{c}} - Bi\right)\right)} - \left\{1 - \exp\left[\sqrt{\psi(s)}\right]\right\} \tag{22}$$

In this case the dimensionless temperature in the superconducting zone is zero as can be revealed from Eq. 21 for $\xi = 0$.

The inverse transformation of Eq. 22 is found to be

$$\theta_1(0, \tau) = \begin{cases} \frac{\varphi}{\tau_i}\psi_1(\tau) & \text{for } \tau < \sqrt{\tau_q} \\ \frac{\varphi}{\tau_i}\left(\psi_1(\tau) - \psi_2(\tau)\right) & \text{for } \tau \geq \sqrt{\tau_q} \end{cases} \tag{23}$$

where

$$\psi_1(\tau) = \frac{1}{\left(s - \left(\frac{G}{1+\theta_c} - Bi\right)\right)} - \exp\left[\left(\frac{G}{1+\theta_c} - Bi\right)\tau\right]\left\{1 - \exp\left[\left(\frac{G}{1+\theta_c} - Bi\right)\tau_q\right]\right\} \tag{24a}$$

$$\begin{aligned}\psi_2(\tau) = {} & \exp\left(\frac{\left(\frac{G}{1+\theta_c} - Bi\right)(\tau_q - 1)}{2\sqrt{\tau_q}}\right)\psi_1(\tau - \sqrt{\tau_q}) \\ & + \left(\frac{\left(\frac{G}{1+\theta_c} - Bi\right)(\tau_q - 1)}{2\sqrt{\tau_q}}\right) \\ & \times \int_{\sqrt{\tau_q}}^{\tau_a} \psi_1(\tau - y)\left(\exp\left((-\frac{G}{1+\theta_c} - Bi)\tau_q\right) - 1\right)\frac{y}{2\tau_q} \\ & \times I_1\left(-\frac{G}{1+\theta_c} - Bi\right)\tau_q\sqrt{(y^2 - \tau_q)}\end{aligned} \tag{24b}$$

and τ_a is defined as

$$\tau_a = \begin{cases} \tau & \text{for } 0 < \tau \le \tau_i \\ \tau_i & \text{for } \tau > \tau_i \end{cases} \tag{25}$$

For the parabolic heat conduction case ($\tau_q = 0$), Eq. 22 can be written as

$$\bar{\theta}_1(0, s) = \frac{\varphi \bar{u}(s)\left\{1 - \exp\left[-\sqrt{\left(s - \left(\frac{G}{1+\theta_c} - Bi\right)\right)}\right]\right\}}{\tau_i\left(s - \left(\frac{G}{1+\theta_c} - Bi\right)\right)} - \tag{26}$$

Thus, the inversion of the temperature field in the parabolic heat conduction case can be computed by taking the inverse of Eq. 26 as

$$\theta_1(0, \tau) = \frac{\varphi}{\tau_i}\int_0^{\tau_a} - \exp\left[\left(\frac{G}{1+\theta_c} - Bi\right)(\tau - y)\right]\mathrm{erf}\left[0.5\sqrt{\tau - y}\right]\mathrm{d}y \tag{27}$$

In this investigation, if the minimal value of the maximum transient temperature for $\tau \ge \tau_i$ equals the current sharing temperature, then the critical energy of the conductor is assumed to be equal to the disturbance energy.

Therefore, the critical energy of the conductor can be computed, for the parabolic heat conduction model, as

$$\varepsilon_p = \frac{\tau_i \theta_{cs}}{\int_0^{\tau_i} \exp\left(\frac{G}{1+\theta_c} - Bi\right)\left[(\tau_m - y)\mathrm{erf}\left[0.5(\tau_m - y)^{-0.5}\right]\right]\mathrm{d}y} \tag{28a}$$

For the hyperbolic heat conduction model,

$$\varepsilon_w = \frac{\tau_i \theta_{cs}}{f_1(\tau_m) - f_1(\tau_m)} \tag{28b}$$

where $\theta_1(0, \tau \geq \tau_i) = \theta(0, \tau_m) = \theta_{cs}$.

3 Results and Discussion

The critical energy was computed analytically based on the analysis of the normal zone propagation in the conductor for selected values of the main factors affecting the problem under consideration, such as the Ohmic heat generation, the current sharing temperature, the duration time of the disturbance, and the thermal relaxation time.

Figure 2 shows the influence of Ohmic heating sources G on the transient response of the superconducting-tape maximum temperature θ_1 based on the hyperbolic model and the associated diffusion model. For $G = 0.2$, ϕ is less than the critical energy. For $G = 1$, ϕ is equal to the critical energy, while ϕ is greater than the critical energy for $G = 8$. The present wave behavior of the maximum temperature displayed in Fig. 2 has three different characteristics in three time intervals. For $\tau \leq \tau_i$ during the

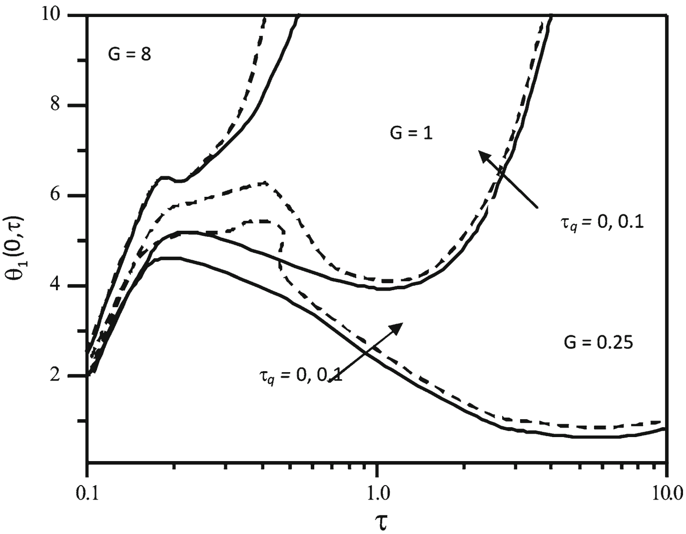

Fig. 2 Influence of Ohmic heat generation (G) on maximum temperature in the normal zone, time variations for $Bi = 0.1$, $\phi = 0.5$, $\tau_i = 0$

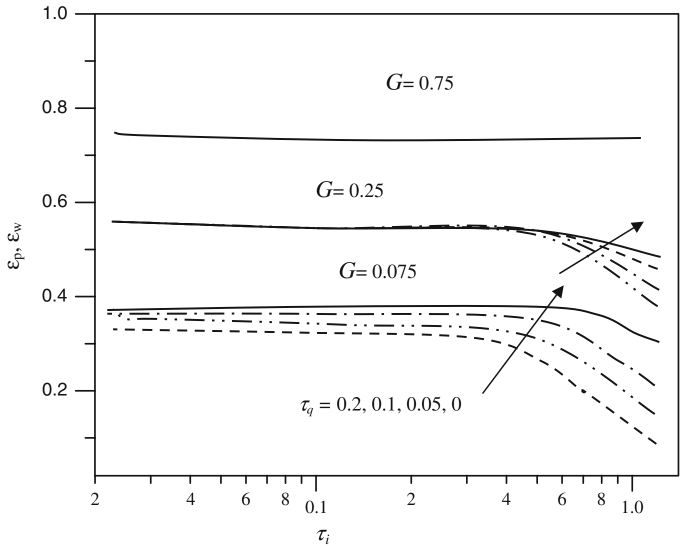

Fig. 3 Variation of critical energy with heat disturbance duration time for different values of Ohmic heat generation and relaxation times for $Bi = 0.1$, $\phi = 0.5$, $\theta_{\rm cs} = 0.15$

disturbance influence, the maximum conductor temperature is changed essentially due to heat absorption. In this time interval, the effect of heat absorption and the difference between the predictions of the two considered models is negligible. For a time of the order of $\tau = O(\sqrt{\tau_q})$, the predictions of the two models are diverging. For large values of time, there are two distinguishing behaviors, one for small G and the other for large G. For small G both temperatures coincide. However, for a large value of G, the difference between the predictions of the two models increases monotonically with time.

The variation of the critical energy with the heat disturbance duration time for different values of Ohmic heat generation and relaxation times is illustrated in Fig. 3. It can be seen that the critical energy is constant until a certain value of τ_i is reached (about 0.4), then a decreasing behavior is observed. Furthermore, the hyperbolic model predicts a lower critical energy compared to the classical heat diffusion (parabolic) model. Furthermore, it is clear that the difference between the two models is more noticeable for larger values of G.

Figure 4 displays the dependence of the critical energy on the current sharing temperature for different values of Ohmic heat generation and relaxation times for $Bi = 0.1$, $\phi = 0.5$, and $\tau_i = 0.25$. It is obvious that the critical energy increases as the current sharing temperature decreases. This can be attributed to the increase of heat generation as $\theta_{\rm cs}$ decreases, because for smaller $\theta_{\rm cs}$, more current passes through the conductor part. In addition, the difference between the predictions of the two models is more noticeable for a higher value of G and increases as $\theta_{\rm cs}$ decreases.

The dependence of the critical energy on Ohmic heat generation for different values of the current sharing temperature and relaxation times is shown in Fig. 5. It is clear

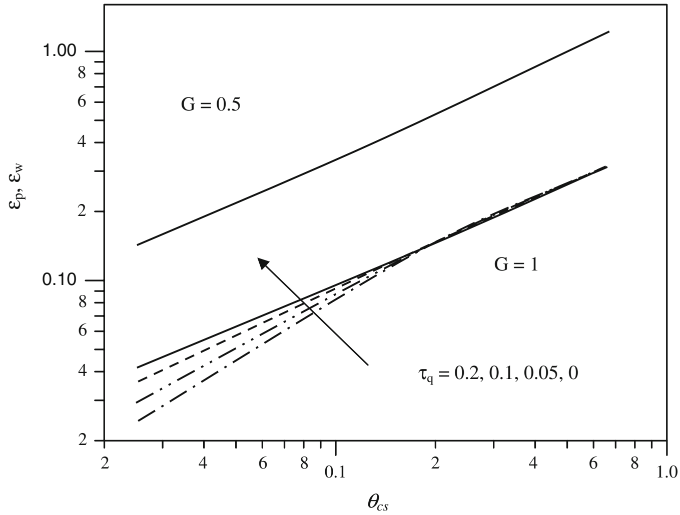

Fig. 4 Dependence of critical energy on heat current sharing temperature for different values of Ohmic heat generation and relaxation times for $Bi = 0.1$, $\phi = 0.5$, $\tau_i = 0.25$

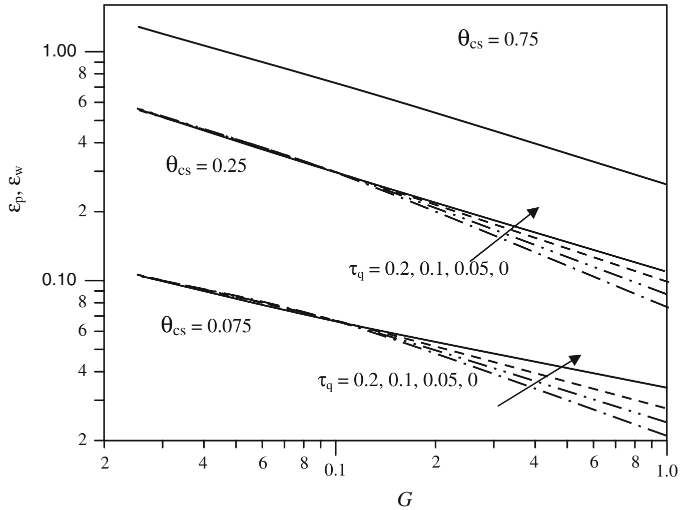

Fig. 5 Dependence of critical energy on Ohmic heat generation for different values of current sharing temperature and relaxation times for $Bi = 0.1$, $\phi = 0.5$, $\tau_i = 0.25$

that the critical energy decrease noticeably with G. Moreover, the difference between the two models is significant for smaller values of the current sharing temperature.

Figure 6 demonstrates the influence of the Biot number on the critical energy at different values of the Ohmic heat generation and relaxation times. The critical energy increases drastically by increasing the Biot number. This is due to the increased convective cooling effect. The convective cooling (Biot number) is an effective way that can be used to increase the critical energy (thus increase the thermal stability) of

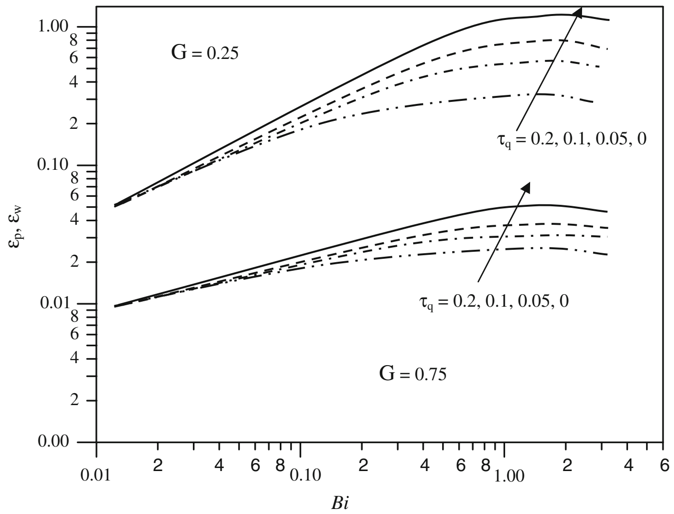

Fig. 6 Dependence of critical energy on Biot number for different values of Ohmic heat generation and relaxation times for $Bi = 0.1$, $\phi = 0.5$, $\tau_i = 0.25$, $\theta_{\text{cs}} = 0.3$

superconductors. Additionally, the difference between the predictions of the two models increases as the Biot number increases.

The effect of the disturbance duration time on the percentage difference between the critical energies calculated based on the two models is depicted in Fig. 7. The percentage difference between the critical energies is defined as $\Delta\varepsilon = (\varepsilon_{p-}\varepsilon_w/\varepsilon_p) \times 100$. This effect has a complex characteristic. $\Delta\varepsilon$ increases with duration time until it reaches a maximum, then it decreases. In addition, the maximum $\Delta\varepsilon$ decreases as the relaxation time decreases (as the model attains the classical conduction model). The rate of increase in the percentage difference is greater for higher values of the relaxation time.

Figure 8 shows the influence of the Biot number on the percentage difference between critical energies calculated based on the two models. The percentage difference $\Delta\varepsilon$ increases as the Biot number increases. In addition, $\Delta\varepsilon$ decreases as the relaxation time decreases. Therefore, for superconductors subjected to severe cooling conditions, the use of the hyperbolic model becomes essential.

4 Concluding Remarks

An analytical model to compute the critical energy of cooled composite superconductors based on the hyperbolic conduction model is presented. The present model takes into account the temperature dependence of the Ohmic heat generation, the finite speed of heat transfer, and the finite duration and finite length of the thermal disturbances. This study show that the critical energy increases as both the current sharing temperature and Biot number increase. Whereas, increases in the Ohmic heat generation and the disturbance duration time result in a decrease in the superconductor critical energy.

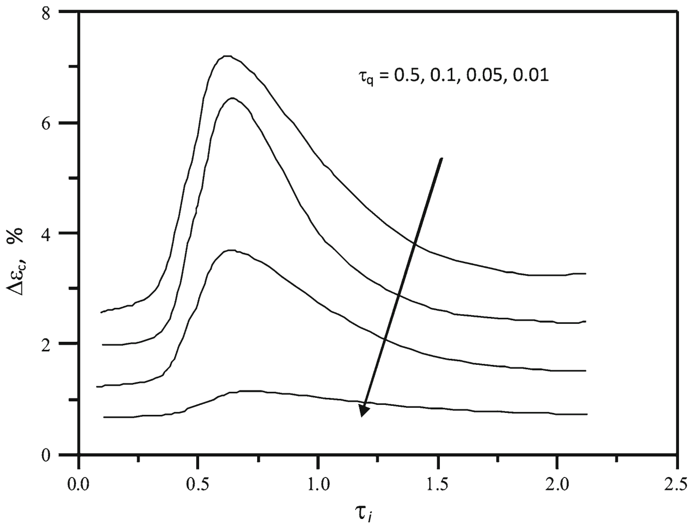

Fig. 7 Effect of disturbance duration, time on percentage difference of critical energy, calculated based on hyperbolic model for $G = 1$, $Bi = 0.2$, $\phi = 0.5$, $\tau_i = 0.25$, $\theta_{cs} = 0.3$

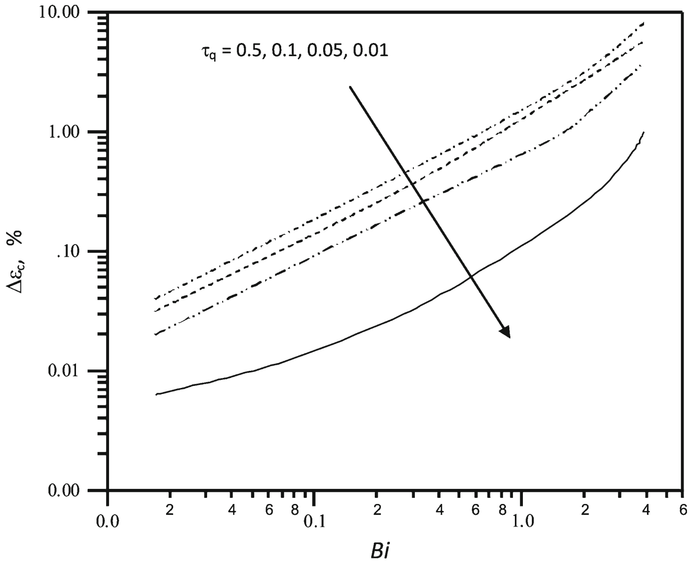

Fig. 8 Influence of Biot number on percentage difference of critical energy calculated based on hyperbolic model for $G = 1$, $\tau_i = 0.2$, $\phi = 0.5$, $\tau_i = 0.25$, $\theta_{cs} = 0.3$

It is found that the hyperbolic conduction model predicts a lower critical energy as compared to the predictions of the parabolic heat conduction model. The difference between the predictions of the two models increases as the current sharing temperature increases. Furthermore, the difference between the predictions of the two models increases as the relaxation time, Ohmic heat generation, and Biot number increase.

Nomenclature

a	Constant given in Eq. 6
A	Conductor cross sectional area, m^2
C	Heat capacity, ρc_P ($J \cdot m^{-3} \cdot K^{-1}$)
d	Superconductor diameter
D	External thermal disturbance
E	Energy of heat disturbance, J
f	Volume fraction of the stabilizer in conductor
$g(T)$	Ohmic Joule heating per unit lateral surface of conductor, $W \cdot m^{-2}$
G	Dimensionless Joule heating source
$g_{\max}$	Maximum Joule heating with the whole current in the stabilizer, $W{\cdot}m^{-3}$
h	Convective heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
I	Transport current, A
J	Current density, $A \cdot m^{-2}$
k	Thermal conductivity of conductor, $W \cdot m^{-1} \cdot K^{-1}$
l	Width of conductor subjected to heat disturbances, m
P	Wetted perimeter
q	Conduction heat flux, $W \cdot m^{-2}$
t	Time, s
T	Temperature, K
T_0	Initial temperature or ambient temperature, K
T_c	Critical temperature, K
T_{cs}	Current sharing temperature, K
t_i	Duration time of disturbance, s
t_q	Relaxation time of heat flux, s
w	Speed of the temperature wave, $m \cdot s^{-1}$
x	Co-ordinate defined in Fig. 1, m

Greek Symbols

α	Thermal diffusivity
$\Delta\varepsilon$	Percentage difference between the critical energies predicted by the two models
ε_p	Critical energy calculated based on parabolic heat conduction model
ε_w	Critical energy calculated based on hyperbolic heat conduction model
ϕ	Dimensionless energy of heat disturbance
τ	Dimensionless time
τ_i	Dimensionless duration time
θ	Dimensionless temperature
θ_{cs}	Dimensionless current sharing temperature
θ_1	Dimensionless maximum temperature
ξ	Dimensionless x-variable
ρ_o	Stabilizer electrical resistivity, Ω
τ_I	Dimensionless duration time of disturbance
τ_q	Dimensionless relaxation time of heat flux

Subscripts

0	Ambient
1	Disturbed
2	Non-disturbed zones
c	Critical zone
cs	Current sharing
max	Maximum

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